THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Engineering and Technology

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

DIPLOMA IN TECHNOLOGY
ELECTRICAL POWER ENGINEERING
MECHATRONICS
TELECOMMUNICATION & INFORMATION ENG.

ENGINEERING MATHEMATICS VI

SEMESTER VI FINAL EXAMINATIONS

SERIES: AUGUST/SEPTEMBER 2011

TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination

- *Question paper*
- *Answer booklet*
- *A Non-programmable calculator*
- *SMP Table*
- *Abridged Laplace Transforms table*

Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Find an attached copy of Abridged Laplace Transform Table
This paper consists of **THREE** printed pages

Question 1 (Compulsory)

a) Given $\begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}$ and the characteristic polynomial $f(\lambda) = \det(A - \lambda I)$, solve the characteristic equation $f(\lambda) = 0$ and hence show that $f(A) = 0$ where 0 = Zero matrix. (7 marks)

b) Determine the eigenvalues of the following matrix.

$$A = \begin{pmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{pmatrix}$$

(5 marks)

c) (i) Determine the Laplace transform of $t^2 e^{-2t} \cos t$ (7 marks)

$$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$

(ii) Given the function

i) Express the function in terms of unit step (Heaviside) function

ii) Determine its Laplace transform (4 marks)

d) (i) Show that the function $e^x(\cos y + j \sin y)$ is analytic function.

(ii) Determine the derivative of the function in d(i) (7 marks)

Question 2

a) Given the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$$

Determine

i) Eigen value of A

ii) Eigenvectors of A

iii) Diagonalize the matrix A such that $P^{-1}AP = S$ where S is a diagonal matrix

(14 marks)

$$C = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$$

b) Determine the eigenvectors of the matrix given (6 marks)

Question 3

$x^2 - y^2 + 2y$

a) Given the function

i) Show that the given function is harmonic (3 marks)

$Z = W^3$

ii) Show that the function remains harmonic under the transformation (5marks)

$U - V = (x - y)(x^2 + 4xy + y^2)$ $f(z) = u + jv$ $z = x + jy$

b) Given and is an analytic function of

$f(z)$

determine in terms of *z* (2 marks)

Question 4

$$\frac{1 - \cos t}{t}$$

a) Determine the Laplace transform of (6 marks)

b) Given the function

$$f(t) = \begin{cases} \sin 2t, & 2\pi < t < 4\pi \\ 0, & otherwise \end{cases}$$

$f(t)$

i) Represent in terms of unit function

ii) Hence, determine its Laplace transform (6 marks)

$$f(t) = \begin{cases} \sin wt & for \quad 0 < t < \pi/w \\ 0 & for \quad \pi/w < t < 2\pi/w \end{cases}$$

c) The Half wave rectifier function is given by

$f(t)$

determine the Laplace transform of

Question 5

$w = \frac{1}{z},$ $x^2 - y^2 = 1$

a) Show that under the transformation the image of the hyperbola is the

$R^2 = \cos 2\phi$

lemniscate (6 marks)

b) The vertices of a triangle in the z- plane is given by *i, 1+i* and *1-i*

i) Determine its image under the transformation $w = e^{5\pi i/3} \cdot z - 2 + 4i$ (6 marks)

ii) On a graph paper plot the triangle on the z-plane and its image on the w-plane.

(5 marks)

c) Transform the curve $x^2 - y^2 = 4$ under the mapping $w = z^2$ (3 marks)