TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSISCS

DIPLOMA IN MEDICAL ENGINEERING

AMA 2351: ENGINEERIGN MATHEMATICS VI

END OF SEMESTER EXAMINATION
SERIES: DECEMEBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination

- *Answer Booklet*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Determine the half-range Fourier cosine series of the function:

$$f(t) = t^2 \; 0 \le t \le \pi$$

(8 marks)

b) Use Taylors series to show that if $\left(x - \frac{\pi}{4}\right)$ is so small that $\left(x - \frac{\pi}{4}\right)^2$ and higher powers may be neglected, then

$$\tan x \approx 1 - \frac{\pi}{2} + 2x$$

(4 marks)

c) Use Maclauria theorem to determine the power series of the function $f(x) = 2x^3 - e^{-3x}$ as far as the term x^4. **(6 marks)**

d) Use Newton Raphson method to approximate $\sqrt{7}$ starting with $x_0 = 2.5$. Give the answer correct to 6 d.p **(8 marks)**

e) Define the following terms:

(i) Interpolation

(ii) Extrapolation **(4 marks)**

Question Two

a) A function f(x) is defined as

$$f(x) = \begin{cases} 0, -5 \le x \le 0 \\ 5, 0 \le x \le 5 \\ f(x+10) \end{cases}$$

(i) Sketch the function for at least three periods

(ii) State whether the function is odd, even or neither

(iii) Determine the Fourier series **(9 marks)**

b) (i) Determine the Maclaurian series of $\ln\left\{\frac{1+x}{1-x}\right\}$

(ii) Hence evaluate ln 3 **(11 marks)**

Question Three

a) (i) Obtain the first four non-zero terms in Maclaurin series of $\cos \lambda x$, where λ is a constant.

(ii) Hence show that

$$\cos \lambda x dx \approx 1 - \frac{\lambda^2}{6} + \frac{\lambda^4}{120} - \frac{\lambda^6}{5040}$$

(9 marks)

b) (i) Expand lnx in Taylors series about the point x = 2 as far as the fifth term. **(6 marks)**

(ii) By putting $x = \frac{1}{2}$ in the result (i), determine an approximate value for ln 2 correct to three d.p **(5 marks)**

Question Four

A periodic function $f(x)$ represents an electromotive force in an electric circuit.

$$f(x) = \begin{cases} \frac{4}{\pi}x + 4 & -\pi < x < 0 \\ \frac{-4}{\pi}x + 4 & 0 < x < \pi \end{cases}$$

a) Sketch the function for at least three periods. **(2 marks)**

b) State whether the function is odd, even or neither. Give reason for your answer. **(2 marks)**

c) Determine the Fourier series for the function. **(10 marks)**

d) By using a suitable substitution and the series above, show that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ **(6 marks)**

Question Five

a) Given that x_n is an approximation to the root of the equation $x^3 - 2x^2 + 2 = 0$. Show using Newton-Raphson method that an approximation x_{n+1} is given by:

$$x_{n+1} = \frac{2x_n^{\ 3} - 2x_n^{\ 2} - 2}{3x_n^{\ 2} - 4x_n}$$

hence by taking $x_o = -0.85$, find to five decimal places the root of the equation. **(8 marks)**

b) Given the table below, use Newton-Gregory interpolation formula to determine:

(i) f(-3)

(ii) f(4)

x	-2	-1	0	1	2
f(x)	-10	0	4	8	18