TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSISCS

DIPLOMA IN ELECTRICAL POWER ENGINEERING (DEPE)
DIPLOMA IN INSTRUMENTATION & CONTROL ENGINEERING (DICE)

AMA 2351: ENGINEERIGN MATHEMATICS VI

END OF SEMESTER EXAMINATION
SERIES: DECEMEBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Given the following matrices:

$$A=\begin{pmatrix}3 & 4 & 0\\ -2 & 6 & -3\\ 7 & -4 & 1\end{pmatrix},\quad B=\begin{pmatrix}1 & 0 & 3\\ 2 & 1 & 2\\ 1 & 3 & 1\end{pmatrix}\quad C=\begin{pmatrix}2\\ 5\\ -1\end{pmatrix}$$

Determine the following;

(i) B^T **(1 mark)**
(ii) B x C **(3 marks)**
(iii) B + A **(2 marks)**
(iv) $|B|$ **(2 marks)**

b) Evaluate $\int_0^1 dx \int_0^x e^{y/x}\,dy$ **(4 marks)**

c) (i) Show that the force field $\vec{F}$ given by $\vec{F}=\left(y^2z^3-6xz^2\right)\vec{i}+2xyz^3\,\vec{j}+\left(3xy^2z^2-6x^2z\right)\vec{k}$ is conservative. **(4 marks)**

(ii) Evaluate $\oint_C \left(x^2+y^2\right)dx+2xydy$, where C is the boundary traversed counter clockwise of the region $R=\left\{(x,y):0\le x\le 1,\ 2x^2\le y\le 2x\right\}$ **(4 marks)**

Question Two

a) Matrix A is defined as:

$$A=\begin{pmatrix}1 & 2 & -3\\ 0 & 3 & 2\\ 0 & 0 & -2\end{pmatrix}$$

determine the eigenvalues of A **(3 marks)**

b) Diagonalize the matrix $B=\begin{pmatrix}6 & -3\\ 2 & 1\end{pmatrix}$ **(14 marks)**

c) Determine the rank of matrix:

$$C=\begin{pmatrix}-1 & 2 & 2\\ 0 & 0 & 1\\ -1 & 2 & 0\end{pmatrix}$$

(3 marks)

Question Three

a) Evaluate $\iint_A xydxdy$ where A is the domain bounded by x axis ordinate x = 2a and the curve $x^2 = 4ay$ **(11 marks)**

b) Determine the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$ **(9 marks)**

Question Four

a) A fluid motion is given by:

$$\vec{V} = (y\sin z - \sin x)\vec{i} + (x\sin z + 2yz)\vec{i} + (xy\cos z + y^2)\vec{k}$$

(i) Show that the motion is irrotational. **(4 marks)**

(ii) Determine the velocity potential **98 marks)**

b) Determine the flux out of the portion of the cylinder $x^2 + z^2 = 4$ in the first octant bounded by x = 0, x = 3 y = 0 and z = 0, given that the vector field $\vec{F} = x\vec{i} + 2z\vec{j} + y\vec{k}$ **(8 marks)**

Question Five

a) (i) Given the matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$, evaluate A^6 **(8 marks)**

(ii) Given $B = \begin{pmatrix} \sin t & t^2 \\ 1 & e^{2t} \end{pmatrix}$ determine $\frac{dB}{dt}$ **(2 marks)**

b) Evaluate $\int_C (x^2ydx + x^2dy)$ where C is the boundary described counter clockwise of the triangle with vertices (0, 0), (1, 0), (1, 1) using Green's theorem. **(10 marks)**