



**TECHNICAL UNIVERSITY OF MOMBASA**  
**Faculty of Applied & Health**  
**Sciences**

DEPARTMENT OF MATHEMATICS & PHYSISCS

**DIPLOMA IN MEDICAL ENGINEERING**

AMA 2351: ENGINEERING MATHEMATICS VI

**END OF SEMESTER EXAMINATION**

**SERIES: APRIL 2015**

**TIME ALLOWED: 2 HOURS**

**Instructions to Candidates:**

You should have the following for this examination

- *Answer Booklet*
- *Mathematical Table*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown  
 This paper consists of **THREE** printed pages

**Question One (Compulsory)**

- $f(x) = \sin x$
- a) (i) Use Maclaurin's series to expand  $f(x)$  into power series in  $x$ .  
 $g(x) = \sin jx = \sin jx$
- (ii) Use the series of  $f(x)$  to show that **(7 marks)**

- b) Write the polynomial  $f(x) = x^4 - 5x^3 + x^2 - 3x + 4$  in ascending power of  $x - 4$  **(6 marks)**

- c) Given that  $x_n = 1.2345, x_{n+1} = 1.2447, x_{n+2} = 1.3124, x_{n+3} = 1.3233$   
 $f(x_n) = 12.5674, f(x_{n+2}) = 13.9831$

Use Linear interpolation and extrapolation to determine  $f(x_{n+1})$  and  $f(x_{n+3})$  correct to 5 d.p. **(6 marks)**

- d) A polynomial function is defined by the following set of functions:

x	2	4	6	8	10
f(x)	-7	9	97	305	681

Use Newton-Gregory formula to evaluate  $f(4.8)$  **(5 marks)**

- e) Evaluate the following integrals.
- (i)  $\int_0^2 \int_0^1 (x^2 + y^2) dy dx$  **(3 marks)**
- (ii)  $\int_0^3 \int_0^{\ln x} dy dx$  **(3 marks)**

**Question Two**

- a) Use Maclaurian theorem to derive the power series of  $f(x) = \sinh x$  **(5 marks)**

$$f(x) = \ln \left\{ \frac{1+x}{1-x} \right\}$$

- b) (i) Given that  $f(x) = \ln \left\{ \frac{1+x}{1-x} \right\}$ . Use Maclaurin theorem to determine the power series of upto  $x^7$   
 (ii) Use your expansion to determine the value of  $\ln 3$  and  $\ln 1.5$  correct to 4 s.f. Hence deduce the value of  $\ln 2$  **(15 marks)**

**Question Three**

$$\int_0^3 \int_0^{\sqrt{9-x^2}} x^2 y \, dy dx$$

a) Evaluate the integral (6 marks)

b) Use triple integral to find the volume of the solid bounded by the planes  $z = 0$ ,  $x = 1$ ,  $x = 2$ ,  $y = 0$ ,  $y = 2$  and the surface  $z = 2x^2 + y^2$  (6 marks)

c) Use double integral to find the area enclosed by the parabola  $y = 6x - x^2$  and the line  $y = x$  (8 marks)

#### Question Four

a) (i) Given that  $x_n$  is an approximation to the root of the equation  $x^3 - 6x^2 + 6x - 8 = 0$ , show using Newton-Raphson method that a better approximation  $x_{n+1}$  is given by:

$$x_{n+1} = \frac{2x_n^3 - 6x_n^2 + 8}{3x_n^2 - 12x_n + 6}$$

(ii) Hence by taking  $x_0 = 5$  find the root of the equation correct to 3 d.p. (9 marks)

b) A function  $f(x)$  is defined by:

x	0	1	2	3	4	5	6
f(x)	-1	2	11	38	95	194	347

Use Newton-Gregory forward difference formula to determine the polynomial that fits the data hence estimate:

(i)  $f(4.8)$

(ii)  $f(6.4)$

(11 marks)

#### Question Five

a) State the Maclaurin series

$$y = \sin^{-1}\left(\frac{x}{2}\right)$$

(i) Obtain the Maclaurian series for  $\sin^{-1}\left(\frac{2}{3}\right)$  up to the term  $x^5$ .

(ii) Use the series to approximate  $\sin^{-1}\left(\frac{2}{3}\right)$  correct to 4 d.p.

$$\sin^{-1}\left(\frac{2}{3}\right) = 0.7297$$

(iii) Calculate the error in the approximate value if

**(12 marks)**

b) Given the function  $f(x) = e^{3x} \sin 3x$ , use Maclaurin series to expand  $f(x)$  up to  $x^3$  evaluate  $\int_0^1 e^{3x} \sin 3x$

**(8 marks)**