TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSISCS

DIPLOMA IN ELECTRICAL POWER ENGINEERING
DIPLOMA IN TELECOMMUNICATION & INFORMATION ENGINEERING
DIPLOMA IN INSTRUMENTATION & CONTROL ENGINEERING

AMA 2351: ENGINEERING MATHEMATICS VI

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination

- *Answer Booklet*
- *Mathematical Table*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

$$\underset{\sim}{A} = a_1 \underset{\sim}{i} + +a_2 \underset{\sim}{j} + a_3 \underset{\sim}{k}, \ \underset{\sim}{B} = b_1 \underset{\sim}{i} + b_2 \underset{\sim}{j} + b_3 \underset{\sim}{k}$$

a) Given that

(i) Show that A.B is a scalar quantity **(3 marks)**

$$\underset{\sim 1}{r} = 2\underset{\sim}{i} + +2\underset{\sim}{j} - \underset{\sim}{k}, \ \underset{\sim 2}{r} = 6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$$

(ii) Hence find the scalar product **(2 marks)**

(iii) Determine the angle between $\underset{\sim 1}{r}$ and $\underset{\sim 2}{r}$ **(3 marks)**

b) Evaluate the following integrals:

(i) $\int_{-3}^{3}\int_{0}^{1}\int_{1}^{2}(x + y + z)dx\ dy\ dz$ **(5 marks)**

(ii) $\int_{0}^{\pi}\int_{0}^{a\sin\theta} r\ dr\ d\theta$ **(3 marks)**

c) Find the area enclosed by $y = x^2$ and the line $y - 2x - 3 = 0$ hence sketch the area under the graph. **(4 marks)**

d) Given that

$$A = \begin{bmatrix} -2 & -1 & 0 \\ -6 & 2 & -3 \\ 0 & -1 & 2 \end{bmatrix}$$

Determine the Eigen values and associated Eigen vectors **(10 marks)**

Question Two

a) Given the vector $\underset{\sim}{A} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}, \ \underset{\sim}{B} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k} \quad \underset{\sim}{C} = \underset{\sim}{i} - \underset{\sim}{k}$ Determine:

(i) A.B **(2 marks)**

(ii) $|A \times C|$ **(3 marks)**

(iii) The angle between A and B **(3 marks)**

b) Evaluate the integral using Green's theorem:

$$\int (2x^2 - y^2)dx + (x^2 + y^2)dy$$

where C is the boundary in the x-y plane of the area bounded by x-axis and the semi circle $x^2 + y^2 = 1$ in the upper half of x-y plane **(12 marks)**

Question Three

a) If $A = x^3 y \underset{\sim}{i} + (x+z) y \underset{\sim}{j} + x^2 z^2 \underset{\sim}{k}$ and $\Phi = 2x^2 y + xzy - 4y^2 z^2 - 5$ Determine at (1, 1, 3)

(i) Div A **(3 marks)**

(ii) Grad θ **(3 marks)**

(iii) Curl A **(3 marks)**

b) Given two vectors $A = 3\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k},\ \ B = \underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ Show that $\underset{\sim}{A} \times \underset{\sim}{B}$ is perpendicular to the vector $C = 9\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$

(3 marks)

c) (i) If $f(x,y,z) = xyz - 2y^2 z + x^2 z^2$ determine $div\, grad\, \Phi$ at (2, 4, 1) **(4 marks)**

(ii) Determine a unit normal to the surface $\Phi = 4xz^3 - 3x^2 y^2$ at (2, -1, 2) **(4 marks)**

Question Four

a) Evaluate the following integrals:

(i) $\int_0^1 dx \int_0^2 e^{y/x}\, dy$ **(4 marks)**

(ii) $\int_0^1 \int_0^1 \int_{\sqrt{x2+y2}}^2 xyz\, dz\, dy\, dx$ **(6 marks)**

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 3,\ z = 0$ **(10 marks)**

Question Five

a) Find the Eigen values and corresponding Eigen vectors of the matrix:

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

(8 marks)

b) A linear time in various system is characterized by the vector differential equation:

$$\frac{dx}{dt} = A\underset{\sim}{x}$$ where $$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Find the state transition matrix $\Phi(t)$ of the system **(10 marks)**

$$|A| = \begin{vmatrix} 1-x & 2 \\ 2 & 1-x \end{vmatrix} = 0$$

c) Given determine the value of two singular matrix **(2 marks)**